

1.1 Shortcuts in Computation

1. Quicker Counting Methods Grouping numbers that add up to 5 or 10 $73+74+27+26 = (73+27)+(74+26)$

Round off numbers that are close to 5 or 10 $73+74+27+26 = 75-2+75-1+25+2+25+1$

2. Sum of numbers that form a pattern **For patterns where:** numbers increase/decrease by same value

1. rewrite sum in reverse order underneath
2. pair up and sum
3. sums of pairs are the same
4. Since sums are the same, multiple sum by number of pairs
5. Divide by 2

Example: Find $2+4+6+\dots+78+80$

$$\begin{array}{r} 2+4+6+\dots+78+80 \\ 80+78+\dots+6+4+2 \\ \hline 82 \times 40 = 3280 \\ 3280/2 = 1640 \end{array}$$

$$2+4+6+\dots+78+80 = 1640$$

3. Quicker Multiplication Methods Remember numbers in their expanded form $3526 = 3000 + 500 + 20 + 6$

3.1 Multiples of 10 $30 \times 25 = 3 \times 10 \times 25 = 3 \times 250 = 750$

3.2 Multiples of 5 $25 \times 6 = 5 \times 5 \times 6 = 5 \times 30 = 5 \times 3 \times 10 = 15 \times 10 = 150$

1.2 Number Logic

Properties of Numbers Primes factor of 1 and itself only

Composites factors other than itself

Divisibility Rules Divisibility rule of 2 EVEN ends with 0,2,4,6,8

Divisibility rule of 3 **sum** of its digits divisible by 3

Divisibility rule of 4 **last 2** digits divisible by 4

Divisibility rule of 5 ends with 0 or 5

Divisibility rule of 6 EVEN AND divisible by 3

Divisible by 8 **last 3** digits divisible by 8

Divisible by 9 **sum** of its digits divisible by 9

Divisible by 10 ends with 0

Squared Numbers $N \times N = N^2$ eg $2 \times 2 = 2^2 = 4$

Cubed Numbers $N \times N \times N = N^3$ eg $2 \times 2 \times 2 = 2^3 = 8$

1.3 Developing Patterns and Shortcuts

Factorising Numbers A number is factorised when expressed as a product of **prime** numbers 250
= 2×125
= $2 \times 5 \times 25$
= $2 \times 5 \times 5 \times 5$

Find prime factors

HCF **largest** counting number that divides into both exactly

Highest Common Factor **method** 1. factorise
2. multiply the factors that are **common**
only those factors that have a pair

example HCF of 240 and 924
 $240 = 2 \times 5 \times 2 \times 2 \times 3 \times 2$
 $924 = 3 \times 2 \times 7 \times 11 \times 2$
HCF = $2 \times 3 \times 2 = 12$

LCM Of all the multiples of the 2 numbers, its the **smallest** multiple they have in common



1.3 Developing Patterns and Shortcuts (cont)

Lowest	method	1. factorise
Common		2. multiply the
Multiple		factors that are
		common and
		factors they don't
		have in common

example

LCM of 120 and 140
 $120 = 2 \times 2 \times 2 \times 3 \times 5$
 $140 = 2 \times 2 \times 3 \times 7$
 $LCM = 2 \times 2 \times 3 \times 2 \times 5 \times 7$

Question (find multiples)	Jack, Art, Fran and Megan work as volunteers at the local kennel. Jack gives the dogs baths every 4 days. Art cleans out cages every 6 days. Fran feeds the animals in section b every 2 days. Megan helps the receptionist every 3 days. How many times in 12 weeks will all 4 helpers be at the clinic on the same day?	how to solve Find all the common multiples from 6 days to 84 days (12 weeks) of 4, 6, 2, 3
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Question (LCM)	Two buses leave the terminal at 8am . Bus A takes 60mins to complete its route and Bus B takes 75mins . When is the next time the two buses will arrive together at the terminal (if they are on time)?	how to solve 1. Find LCM of 60 and 75. 2. Add LCM to 8am
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1.3 Developing Patterns and Shortcuts (cont)

Question (HCM and LCM)	Dennis has a choice between two house numbers on Small Street. The two house numbers have their highest common factor of 6. Their least common multiple is 36. One of the house numbers is 12. What is the other number?	how to solve work backwards
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1.4 Logic Deduction

Logic Deduction Problems	If need to add groups of things, use biggest numbers first
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Question	What is the minimum number of coins needed to make \$4.85 from only 5c, 20c, 50c coins	Start with biggest coins first, working through to smaller coins $9 \times 50c = \$4.50$ $1 \times 20c = 20c$ $3 \times 5c = 15c$
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assume worse case scenario

investigate standard case

write relations between numbers down

1.5 Space, Area and Volume

Area of Rectangle	length x width
Area of Triangle	$A = \text{base} \times \text{height} / 2$
Volume of Cube	$V = a^3$ where a is length of a side
Volume of Rectangular Prism	$V = \text{length} \times \text{height} \times \text{depth}$
1m	= 100cm

Finding Area of Rectangular Shapes

Method 1	Divide shape into rectangles	Find area of each and find total
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1.5 Space, Area and Volume (cont)

Method	Extend shape into one larger rectangle	1. Find area of larger rectangle (X) 2. Find area of missing rectangle (Y) 3. Larger rectangle (X) - Missing rectangle (Y)
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1.6 Equations

Pronumerals Boxes to store missing numbers

Letters to represent unknown numbers

Use **x**, **y** and **z**

Rearranging Equations = is like a balancing scale

solving an equation **aim** of finding the unknown number

rearranging equations **how** to solve an equation

how if we do something to one side, we need to do the same thing to the other side

eg. if we **add 3** to one side, we need to **add 3** to the other side

eg. if we **times by 3** to one side, we need to **times by 3** to the other side

+

-

x

/

Simultaneous Equations if there are 2 unknowns, need 2 equations

1.6 Equations (cont)

1. Solving by Adding and Subtracting Equations

example

$$5x - y = 4 \quad (1)$$

$$2x + y = 10 \quad (2)$$

$$(1) + (2)$$

$$7x = 14$$

$$x = 2$$

$$y = 6$$

example

$$7x + y = 18 \quad (1)$$

$$2x + 2y = 12 \quad (2)$$

$$(1) \times 2$$

$$14x + 2y = 36 \quad (1a)$$

$$(1a) - (2)$$

$$12x = 24$$

$$x = 2$$

$$y = 4$$

2. Solving by Substitution

method

- rearrange one equation for y
- substitute y into other equation

example

$$5x - y = 4 \quad (1)$$

$$2x + y = 10 \quad (2)$$

rearrange (1)

$$y = 5x - 4 \quad (1a)$$

substitute (1a) into (2)

$$2x + (5x - 4) = 10$$

$$x = 2$$

$$y = 6$$

Turning word problems into an equation

Step 1 What are the unknowns? Give each a letter, **x**, **y**

Step 2 Find the equations to solve

Step 3 Solve the simultaneous equations

Example Questions

The quotient of two numbers is 4 and their difference is 39. What is the smaller number of the two

The sum of the ages of Alan and Bill is 25; the sum of the ages of Alan and Carl is 20; the sum of the ages of Bill and Carl is 31. Who is the oldest of the three boys and how old is he?

1.7 Probability, Venn Diagrams and Whodunits

1. Certainty Problems

Typical Question	Suppose that there are ten black and ten navy socks in your drawer. Your room is dark and you cannot turn on the light. What is the smallest number of socks that you must take out of your drawer to be certain that you have a pair of the same colour?	Basically, to be certain of "an outcome", what is the smallest number of "actions" required to take
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Strategy Start from smallest and go up

1 sock	can't be certain
2 socks	can't be certain
3 socks	can be certain

2. Certainty Problems with Restrictions

Typical Question	As above question, but what is the smallest number of socks needed to ensure we get a pair of black socks	Restriction is it must be black socks
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Strategy Think Worst Case Scenario

Worst case is you could in 10 picks, pick only Navy socks. 2 more picks you'll be certain to get a pair of black socks

12 socks	can be certain
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Venn Diagrams circle represents **sets** or groups of *things* that are same

1.7 Probability, Venn Diagrams and Whodunits (cont)

Example Question	There are 160 students in Year 5. Of these students, 69 walked to school and 57 caught a train to school. If 148 students either walked to school or caught the train, how many students walked and caught a train to school?
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Draw a Venn diagram with a circle for students that walked and students that caught the train

Where they overlap, are the number of students that walked and caught the train

Whodunits

Strategy	Use a table, with different characteristics in columns and members of a group in rows	Usually the answer needed are the characteristics
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Example Question	Martin, Bill and Dave (members of a group) play first base, second base, and third base (characteristics) on their school softball team, but not necessarily in that order. Martin and the third baseman took Dave to the movies yesterday. Martin does not play first base. Who's on first base?
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1.8 Motions, Books, Clocks and Work Problems

1. Motion Problems

distance = rate x time

Example Question 1 Two trains leave the same station at the same time, but in **opposite** directions. One train averages 56 km/h and the other averages 64 km/h. How far apart will the trains be when three hours have passed?

Strategy Step 1 Whats the distance after 1hr? (Draw a diagram)

$$56\text{km} + 64\text{km} = 120\text{km}$$

$$56\text{km/hr} + 64\text{km/hr} = 120\text{km/hr}$$

Step 2 Whats the distance after 3hrs?

$$120\text{km} \times 3 = 360\text{km}$$

if **opposite** direction,
add

Example Question 2 Suppose that these two trains start from the same station at the same time, this time in the same direction. How far apart will the fronts of the trains be at the end of the three hours?

Step 1 Whats the distance after 1hr? (Draw a diagram)

$$64\text{km/hr} - 56\text{km/hr} = 8\text{km/hr}$$

$$64\text{km} - 56\text{km} = 8\text{km}$$

Step 2 Whats the distance after 3hrs?

$$8\text{km} \times 3 = 24\text{km}$$

if **opposite** direction,
subtract

2. Book Problems

look at the structure of counting numbers used for book pages

1.8 Motions, Books, Clocks and Work Problems (cont)

Example Question A printer uses an old-style printing press and needs one piece of type for each digit in the page numbers of a book. How many 2s will the printer need to print page numbers from 1 to 250

consider the numbers place by place

number of 25
times 2s
appear in
the 1s
place

number of 30
times 2s
appear in
the 10s
place

number of 51
times 2s
appear in
the 100s
place

$$\text{answer} = 25 + 30 + 51 = 106$$

3. Clock Problems

elapse time amount of time that has passed

solve using facts about time

Example Question A certain clock gains one minute of time every hour. If the clock shows the correct time now, in how many hours will it next show the correct time again without regard to am or pm?

Fact 1 A clock that has stopped Will show the correct time every 12 hrs. As it stopped at 6.03am on Monday. It was correct at the time it stopped. It will be correct again, when the time is at 6.03pm



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1.8 Motions, Books, Clocks and Work Problems (cont)

Fact 2 The clock in the problem must gain 12 hours to show correct time again

thus 12 hrs = 60mins x 12
= 720mins

thus as clock gains 1 min in 1 hr the clock will gain 720min in 720hrs

$$720/24=30\text{days}$$

4. Work Problems

solving fractional parts of whole numbers and draw diagrams using

Example Question Paul can do a certain job in 3hrs and John can do the same job in 2hrs. At these rates, how long would it take Paul and John to do this job if they work together

Strategy Step 1 Draw a diagram for Paul and John. Fractional parts done in each hour

Step 2 Using the diagram, in one hour they can complete $\frac{1}{3} + \frac{1}{2} = \frac{5}{6}$ of the job

Step 3 Work out how long to complete job

$\frac{1}{5}$ of job left

$60\text{min} / 5 = 12\text{mins}$ to complete $\frac{1}{5}$ of job

answer = 1hr 12mins

1.9 Problem Solving Strategies

1. Drawing a picture or diagram

Example Question The lengths of three rods are 5cm, 7cm, and 15cm. How can you use these rods to measure a length of 13cm?

2. Making an organised list

1.9 Problem Solving Strategies (cont)

Example Question Five students hold a chess tournament. Each of the students plays each of the other students just once. How many different games are played?

3. Making a table

Example Question Two dice both have faces numbered from 1 through to 6. Suppose that you roll the two dice. What is the probability of rolling a sum of 8 in the uppermost faces?

4. Solving a simpler related problem

Example Question The houses on Thomas Street are numbered consecutively from 1 to 150. How many house numbers contain at least one digit 7?

5. Finding a pattern

Example Question What is the sum of the following series of numbers?

6. Guessing and Checking

Example Question Arrange the counting numbers from 1 to 6 in the circles so that the sum of the numbers along each side of the triangle is 10.

1.10 Problem Solving Strategies

1. Acting out the problem

Example Question Suppose that you buy a rare stamp for \$16, sell it for \$22, buy it back for \$30, and finally sell it for \$35. How much money did you make or lose?

2. Working backwards

Example Question At the end of a school day, a teacher had 15 crayons left. The teacher remembered giving out 13 of all her crayons in the morning, getting 8 back at recess, and giving out 9 crayons after lunch. How many crayons did the teacher have at the start of the day?



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1.10 Problem Solving Strategies (cont)

3. Writing an Equation

Example Question The triple of what number is sixteen greater than the number?

4. Change your approach

Changing your point of view

Are you assuming something that's not in the question

Example Question Draw four continuous line segments through the nine dots

5. Using Reasoning

Example Question A school has 731 students. Prove that there must be at least 3 students who have the same birthday.

6. Miscellaneous

Example Question Three apples and two pears cost 78 cents. But two apples and three pears cost 82 cents. What is the total cost of one apple and one pear?

2.1 Logical Approach to Problem Solving

4 Steps to Problem Solving

Step 1 Understand the problem

Step 2 Develop a plan **choose a problem solving strategy**

Step 3 Carry out the plan

Step 4 Reflect

Mathematical Terms used in the Olympiad

Standard Form 1358

Expanded Form $1 \times 1000 + 3 \times 100 + 5 \times 10 + 8 \times 1$

Exponential Form $1 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 8 \times 1$

Whole numbers 0, 1, 2, 3, ...

Counting numbers 1, 2, 3, ...

Divisibility A is divisible by B, if B divides into A with zero remainder **If so, B is a factor of A**

2.1 Logical Approach to Problem Solving (cont)

Prime number counting number greater than 1, which is divisible only by itself and

Composite number counting number greater than 1 which is divisible by a counting number other than 1 and itself

A number is factored completely when it is a product of prime numbers

Order of Operation BODMAS

common or simple fraction a/b where a and b are whole numbers and b is not zero

unit fraction common fraction with a numerator of 1

proper fraction a/b where $a < b$

improper fraction a/b where $a > b$

complex fraction numerator or denominator contains a fraction

20th century 100 year period 1901-2000 inclusive

average of a set of N numbers sum of the N numbers divided by N

acute angle less than 90 degrees

right angle 90 degrees

obtuse angle greater than 90 degrees

straight angle 180 degrees

reflex angle more than 180 degrees and less than 360 degrees

scalene triangle no equal angles

isosceles triangle 2 equal angles

equilateral 3 equal angles

right-angled 90 angle



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2.1 Logical Approach to Problem Solving (cont)

congruent shapes	shapes on the same plane whose sides and angles are the same
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2.2 Types of Problems

- Translation Problems** translate word sentences to mathematical sentences

Example Question Farmer Joe bought 2 bags of feed for \$4 each and 1 bag of feed for \$3. How much did the feed bags cost altogether?

- Application Problems** 'real-world' problems, usually involve calculations with money, to find **discounts**, **profits** or **cost** of items

Example Question Shop A is offering a 10% discount on 34cm colour TV sets priced normally at \$379. Meanwhile Shop B is offering 15% discount on the same sets priced normally at \$409. Which shop should you purchase the TV from?

- Process Problems** Usually require using general problem solving steps and specific strategies. May use short-cuts when aware of patterns

Example Question The first 4 triangular numbers are 1, 3, 6, 10. What will the 10th triangular number be?

- Puzzle Problems** like riddles

Example Question Three Australian students who were born in different countries have last names Brown, Black and Bright. Their first names are Jim, John and Jane but not necessarily in that order. Using the information below can you determine the full name of each student?

Brown was born in Australia
 Bright has never been to Malaysia
 Jane was born in England
 Jim was born in Malaysia



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